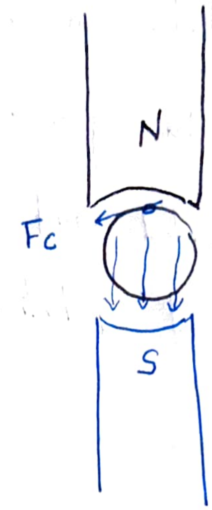
Unit - 2

D.C. Motor Principle :-



When a current carrying conductor is put in a magnetic field. A force is produced on it. The effect of placing a current carrying conductor is put in a magnetic field is shown in Figure.

Let us consider one such conductor is placed in a slot of armature and suppose that it is acted upon M.F. Since the conductor is in slot of the circumference of the rotor. The force Fc acts in the tangential direction of the rotor. Thus a torque is developed in the rotor. Similarly torque are produced on all the rotor conductors. Since the rotor is free to move and it starts rotating in the Anti c.w direction. The torque produced on the rotor is transferred to the shaft of the rotor and can be utilised to drive a mechanical load.

E.M.F eqn of a D.C. machine :- (Generator or motor)

As the armature rotates a voltage is generated in the coil. In case of the generator the emf of the rotation is called the generated emf or armature emf. In case of motor the emf of rotation is called back emf or counter E.M.F.

Let ϕ = useful flux per pole in webers

P = Total no. of poles

Z = Total no. of conductors in the armature

n = speed of rotation of armature in Revolution per second (RPS)

A = No. of parallel path through the armature b/w the brushes of opposite polarities.

(In lap winding $A = P$
In wave " $A = 2$)

$\therefore$ Z/A = No. of armature conductor in series for each parallel path.

Since the flux per pole is ϕ, each conductor cuts a flux "$p\phi$" in one revolution.

Generated voltage / conductor $\Rightarrow \dfrac{\text{flux cut for revolution in weber}}{\text{time taken for one revolution in seconds.}}$

$$\Rightarrow \frac{p\phi}{1/n} \Rightarrow np\phi \qquad \left\{ \begin{array}{l} n\,rev \rightarrow 1\,sec \\ 1\,rev \rightarrow 1/n\,sec \end{array} \right.$$

The generated voltage E is determined by the no. of armature conductors in series in any one path b/w the brushes. Therefore the total voltage generated.

E $\Rightarrow$ Avg voltage per conductor $\times$ No. of conductor in series per path

$$\boxed{E = \frac{np\phi z}{A}}$$ $\rightarrow$ This eqn is called emf eqn of DC ~~motor~~ machine.

$$\left(n = \frac{N}{60} \right)$$

Ques 1 4 pole wave wound armature has 720 conductors and it rotates at 1000 rev/min. If the useful flux is 20 milliweber. Calculate the generated voltage?

→ $P = 4$ $\quad A = 2$ $\quad Z = 720$ $\quad N = 1000$ rpm

$$\phi = 20\,mWb = 20 \times 10^{-3}\,Wb$$

$$E_g = \frac{P\phi Z N}{60A} \Rightarrow \frac{4 \times 200 \times 10^{-3} \times 720 \times 1000}{60 \times 2} = \boxed{480\ \text{Volt}}$$

Ques 2 A D.C generator has armature E.M.F of 100 V when the useful flux per pole is 20 mWb and the speed is 800 rpm. Calculate the generated emf

(a) with the same flux and a speed of 1000 rpm.

(b) with the flux per pole of 24 mWb & speed of 900 rpm

→ ~~(a)~~ $E_1 \Rightarrow 100\,V$ $\quad \phi_1 = 20\,mWb$ $\quad N_1 = 800$ rpm

$$E \propto \phi N$$

$$\frac{E_1}{E_2} = \frac{\phi_1 N_1}{\phi_2 N_2}$$

(a) same flux $\{\phi_1 = \phi_2\}$, $N_2 = 1000$ rpm

$$\frac{E_1}{E_2} \Rightarrow \frac{N_1}{N_2}$$

$$\frac{100}{E_2} \Rightarrow \frac{800}{1000} \Rightarrow E_2 = \boxed{125 \text{ Volt}}$$

(b) $\phi_2 \Rightarrow 24$ mWb $\quad N_2 = 900$ rpm $\quad E_2 \Rightarrow ?$

$$\frac{E_1}{E_2} = \frac{\phi_1 N_1}{\phi_2 N_2}$$

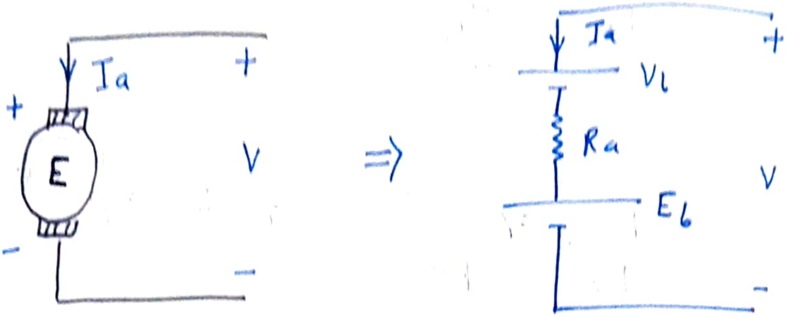
$$\frac{100}{E_2} = \frac{20 \times 800}{24 \times 900} \qquad \boxed{E_2 = 135 \text{ Volt}}$$

Back EMF :- When the motor armature rotates its conductor cut the magnetic flux. Therefore the emf rotation is induced in the conductor. In case of motor the emf rotation is called back EMF or counter E.M.F.

The back emf opposes the voltage. Since the back E.M.F is induced due to the generator action its magnitude is therefore is same as that for generated EMF in dc generator. i.e

$$E = \frac{n P \phi Z}{A} = \frac{N P \phi Z}{60 A}$$

Equivalent circuit of a D.C Motor armature :-



If motor current flows from the line the armature against the generated voltage.

By K.V.L :-

$$\boxed{V = E_b + I_a R_a}$$

V → terminal voltage

E_b → back emf

$I_a R_a$ → armature current & resistance

It is the fundamental motor equation.

If the voltage drops V_b in the brushes also considered then by KVL

$$\boxed{V = E_b + I_a R_a + V_b}$$

Torque eqn of a D.C motor :-

We know, voltage eqn of D.C motor is

$$V = E + I_a R_a \quad \text{— (i)}$$

(i) X $I_a R_a$

$$\underbrace{V \times I_a \cancel{R_a}}_{\text{Electrical power}} = E \times I_a \cancel{R_a} + \underbrace{(I_a^2 R_a)}_{\text{Cu power (loss in the armature)}} \quad \text{— (ii)}$$

$$\text{Input} = \text{Output} + \text{loss} \quad \text{— (iii)}$$

Comparing eq (ii) and (iii)

Mechanical power developed by the armature = Electromagnetic power

T_{av} ⇒ avg. electromagnetic torque developed by the armature in N-m.

Then mechanical power developed by the armature

$$P_m = \omega . T_{avg}$$

$$= 2\pi n . T_{avg}$$

Therefore $P_m = E . I_a = 2\pi n \; T_{avg}$ ——— (iv)

$$I_a E = \frac{n P \phi z}{A} \qquad \frac{n P \phi z}{A} \Rightarrow I_a E$$

$$\boxed{T_{av} = \frac{PZ}{2\pi A}\phi . I_a}$$

Torque eqn of D.C machine

For a given machine P, Z, A are constant

$$\therefore \frac{PZ}{2\pi A} = k = \text{constant}$$

$$\boxed{T_{av} = k . \phi . I_a} \rightarrow (7)$$

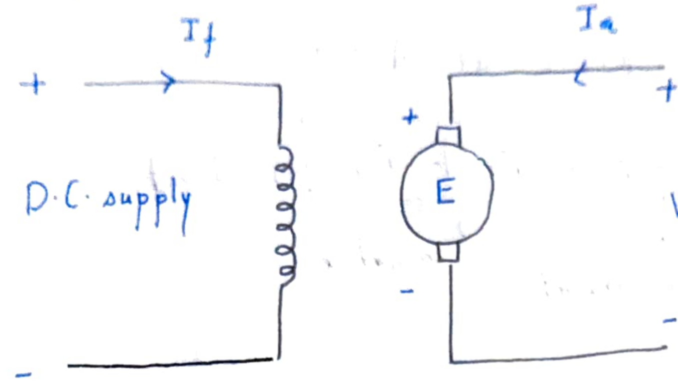
$$T_{av} \propto \phi . I_a$$

Hence the torque developed by D.C motor is directly proportional to flux per pole and armature current.

Types of D.C motor :-

1. Seperately excited D.C motor
2. Shunt wound or shunt D.C motor
3. Series wound or series D.C motor
4. Compound wound or compound motor.

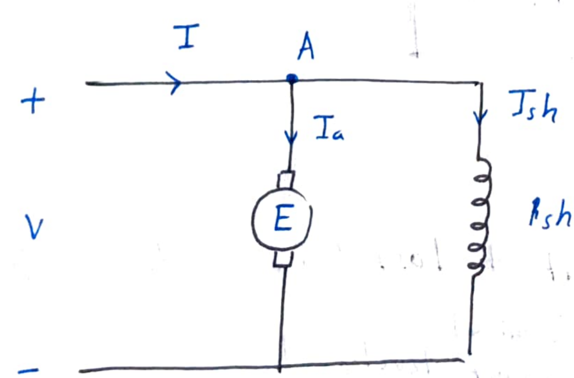
(i) Seperately excited D.C. motor :-



(If it would have been generator than Ia direction reverses)

As the name implies the field coil are energised by sep. D.C. source.

(ii) Shunt Motor :-



In this field winding is connected in parallel with the armature. ~~The cu~~

The current, voltage and power eq for shunt motor are written as follows :-

Current Equation :-

By applying K.C.L at A :-

$$\boxed{\underbrace{I}_{\text{line current}} = \underbrace{I_a}_{\text{armature}} + \underbrace{I_{sh}}_{\text{shunt}}}$$

Voltage equation :-

For field winding circuit :-

$$\boxed{V = I_{sh} R_{sh}}$$

For armature winding circuit :-

$$\boxed{V = E + I_a R_a}$$

Power equation :-

$$P_{in} = P_{out} + \text{loss (in armature \& field)}$$

(mechanical power developed)

$$VI = P_m + I_a^2 R_a + I_{sh}^2 R_{sh} \; (V \cdot I_{sh})$$

$$P_m = V\underbrace{(I - I_{sh})}_{I_a} - I_a^2 R_a$$

$$P_m = (V - I_a R_a) I_a$$

$$\boxed{P_m = E I_a}$$

We know that

$$V = E + I_a R_a$$

Multiply by I_a

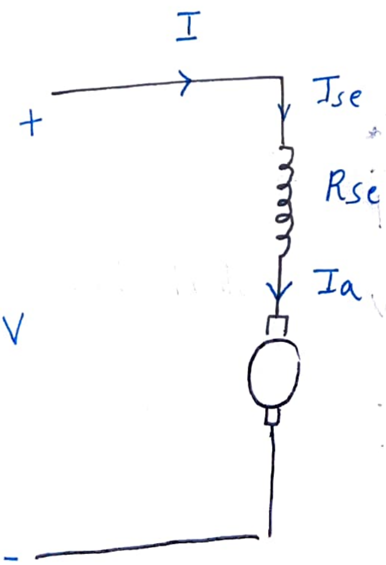
$$I_a V = E \cdot I_a + I_a^2 R_a$$

$$V I_a = P_m + I_a^2 R_a$$

where $V \times I_a$ is the electrical power supplied to the armature of motor.

(iii) Series motor :-

In series, field winding is in series with armature.
connected



Current eqn:-

$$\boxed{I = I_{se} = I_a} \rightarrow (i)$$

Voltage eqn:-

By using KVL

$$V = E + I_a R_a + I_{se} R_{se}$$

$$\boxed{V = E + I(R_a + R_{se})} \rightarrow (ii)$$

Power eqn:-

multiplying eq (ii) by I

$$VI = EI + I^2(R_a + R_{se})$$

Power input = mechanical work + armature loss + field loss

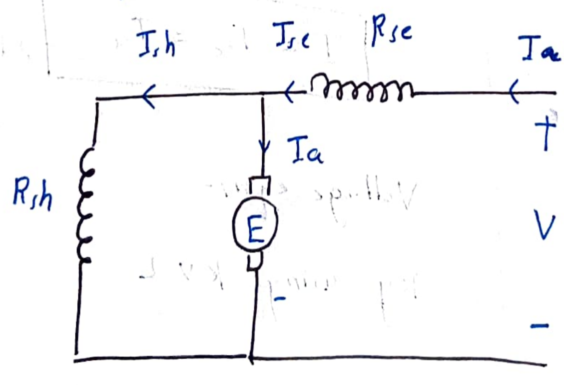
Comparing

$$\boxed{P_m = EI}$$

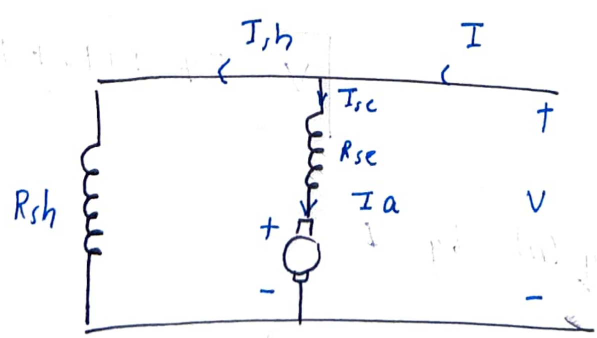
(iv) Compound wound / Compound motor :-

A D.C motor having both shunt and series windings is called a compound motor. It may be short shunt compound motor or long shunt compount motor.

If the shunt field is connected in parallel with the armature alone, the motor is called shunt-short compound motor. If the shunt field is ॥ with the both armature and series field then the motor is called long shunt compound motor.



Short shunt



Long shunt

If the magnetic flux produced by series winding assist the flux produced by the shunt field winding, then it is known as cumulatively compounded motor.

If the series field flux opposes the shunt field flux, then known as differentially compounded motor.

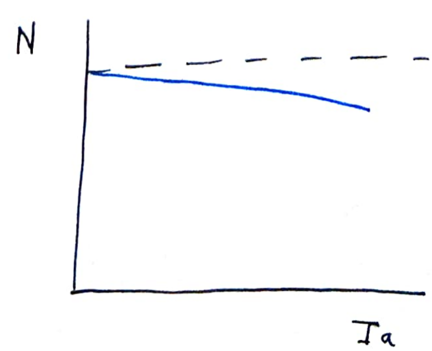
① Characteristics of shunt or seperately excited D.C Motor

In both the cases of shunt and separately excited D.C. motors, the field is supplied from a constant voltage so that the field current is constant.

(a) Speed armature current characteristics :-

The shunt current $I_{sh} = \frac{V}{R_{sh}}$. If V is constant I_{sh} will be constant. Hence flux is constant at no load. The flux decreases slightly due to armature reaction. If the effect of armature rxn is neglected. The flux (ϕ) remains constant. We know that $N \propto \frac{E}{\phi}$

$N \propto \frac{V - I_a R_a}{\phi}$. If ϕ is constant $N \propto V - I_a R_a$

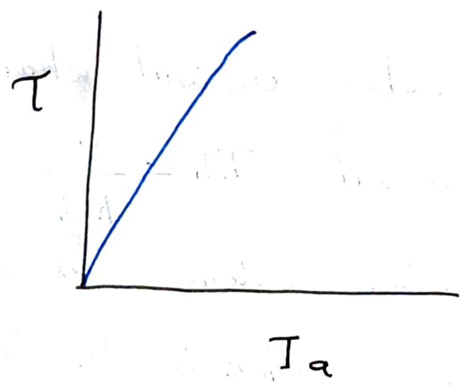


From above eqn shows that the speed of motor decreases linearly with increase in armature current as shown in figure. Since $I_a R_a$ at full load is very small compared to V the drop in speed is no load to full load is very small. Hence for all practical purposes the shunt motor can be taken as constant speed motor.

(b) Torque - Armature current :-

We know that $T \propto \phi . I_a$. If ϕ is constant

$T \propto I_a$



② Characteristic of a D.C. series motor :-

We know that $N \propto \frac{E}{\phi}$ and $N \propto \frac{V - I_a(R_a + R_{se})}{\phi}$

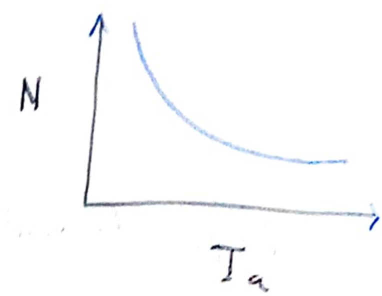
At low value of I_a, the voltage drop $I_a(R_a + R_{se})$ is negligibly small in comparision with V. Therefore $N \propto \frac{1}{\phi}$. In a series motor flux (ϕ) is produced by I_a flowing in the field winding so that $\phi \propto I_a$

Hence series motor is variable flux machine

Therefore $N \propto \frac{1}{I_a}$

In a series motor the speed is inversely propotional to armature current.

(a) Speed- Current Characteristics



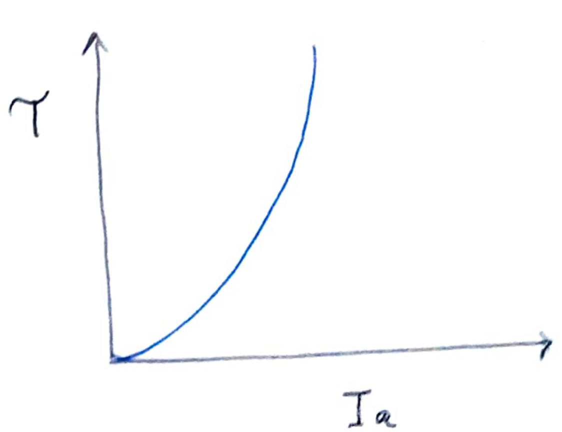
(b)

Torque - Armature current :-

$T \propto \phi I_a$

In d.c series motor $\phi \propto I_a$. So $T \propto I_a^2$

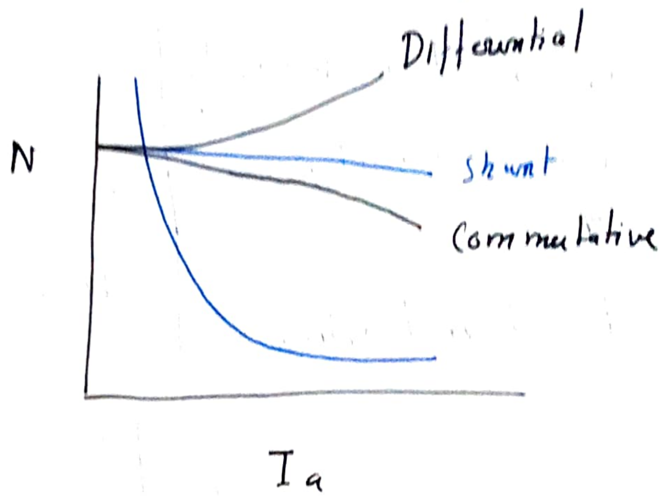
This shows that curve would be parabolic.



(3) Characteristic of Compound Motor.

A Compound motor has both shunt and series field winding. So its characteristics are intermediate b/w shunt & series motor.

(a) Speed - Current Characteristic



(b) Torque - Current characteristics :-